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Asymmetric Stator Interaction Noise

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Noise reduction benefits of nominally cut-off rotor-stator designs are limited in practice by deviations from symmetry in fan construction and by operating airflow irregularities. One type of asymmetry is a set of small random variations from precisely equal spacing in the circumferential stator vane positions. Acoustic tests on a model fan suggested that this asymmetry, due to manufacturing imperfection, was the source of radiated sound at blade passage frequency for a rotor-stator combination designed to be cut-off. An analysis of the fan noise produced by extraneous propagating modes generated by rotor wake interaction with stator vanes that have random vane angle deviations is given here. Comparison of predicted tone levels with model fan data indicates good quantitative agreement. Inferences are made about the effects of stator asymmetry in full-scale engine fans.

Nomenclature

a = speed of sound
$a_{m\mu}$ = axial direction cosine of m, μ wave, $=\sqrt{1-1/\xi^2_{m\mu}}$
$A_{m\mu}$ = power-effective area for m, μ mode
a_m = direction cosine of m-lobe mode $=\sqrt{1-1/\xi^2_m}$
B = number of rotor blades
c = designation for c_m when modes have equal amplitudes
c_m = amplitude of mth mode due to rotor interaction with single vane
C_m = amplitude of mth mode due to rotor interaction with stator assembly
$E(\ )$ = expected value of operand
$f(r_m)$ = probability density function for r_m
$g(\rho_m)$ = probability density function for ρ_m
$g'(\rho'_m)$ = probability density function for ρ'_m
$h(U)$ = probability density function for U
$H(U)$ = probability distribution function for U, $=\int_{-\infty}^{U} h(u)\,du$
$H'(U)$ $=1-H(U)$
i $=\sqrt{-1}$, also used as index
j = index
k = index
m = circumferential mode index
M = circumferential mode index for symmetric stator interaction
n = harmonic index
N = number of propagating circumferential modes
p_m $=nB-m$
p = local duct pressure due to rotor interaction with stator assembly
p' = local duct pressure due to rotor interaction with single stator vane
q = stator vane index
r_m = normalized modal coefficient, $=|C_m/c|$
Re = real part of operand
t = time coordinate
U = normalized power, $=\sum_m a_m |C_m/c|^2$
$\bar{U}$ $=E(U)$
V = number of stator vanes
W = acoustic power
W' = approximation for acoustic power, $=\sum_m a_m |C_m|^2$
α_q = angular location of qth vane
β_q = corresponding location in symmetric stator, $=q\,2\pi/V$
β_{qj} $=\beta_q-\beta_j$
$\delta_{p_{mv}}$ = Kronecker delta function = 1 for p_m an integer multiple of V; = 0 otherwise
ϵ_q = deviation of qth vane from nominal angular location $=\alpha_q-\beta_q$
ϵ_{qj} $=\epsilon_q-\epsilon_j$
θ = pressure field angular coordinate
μ = radial mode index
$\xi_{m\mu}$ = cutoff ratio of m,μ mode
ξ_m = cutoff ratio of m-lobe mode
ρ = density of medium
ρ_m = square of normalized modal coefficient, $=r_m{}^2$
ρ'_m = normalized power component, $=a_m\rho_m$
σ_ϵ = standard deviation of set of angular deviations, ϵ_q
σ'_ϵ = normalized standard deviation, $=\sigma_\epsilon/(2\pi/V)$
$\sigma^2_{p_m}$ = variance defined as $=\frac{1}{2}p^2_m\sigma^2_\epsilon V$
σ^2_m = variance defined as $=a_m\sigma^2_{pm}$
Ω = rotor shaft angular velocity
$*$ = complex conjugate
★ = convolution

I. Introduction

THE principal source of subsonic tip speed turbomachinery noise at blade frequency harmonics is the interaction between the flowfields of rotating blades and stationary vane elements. When the assembly of V stator vanes is perfectly symmetrical the modes associated with any harmonic nB of shaft speed are limited[1] to those for which the circumferential mode index m is given by $m=mB\pm kV, k=0,1,2...$ By selecting a sufficiently large number of stator vanes, the least absolute value of m generated by the above expression can often be made large enough so that this mode (and the others as well) is "cut-off" and will decay inside the powerplant ducting instead of propagating to the exterior. This result has been applied extensively to the design of modern aircraft powerplants.

In practice there may be several deviations from ideal conditions so that blade-passage noise usually can be detected. Such deviations include vane-to-vane differences in aerodynamic response and angular spacing (circumferentially) due to manufacturing tolerances in the stator

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construction. Recent acoustic wind tunnel test results (see Sec. II) on a model aircraft fan suggested that stator irregularities were the source of measured blade frequency noise for "cut-off" designs: As axial spacing between rotor and stator was changed the radiated blade frequency noise varied, despite the fact that the rotor-stator interaction was predicted to be cut-off. These results motivated the subject investigation which sought to answer the question: How do stator vane imperfections affect noise generation for a cut-off rotor-stator combination? A secondary question concerns the possibility of employing deliberate stator asymmetry to reduce noise in a configuration having a propagating primary interaction.

II. Background Tests

The current analytical study was motivated by experimental results that were obtained on a 4.1 in. diameter model fan in an installation referred to as the Advanced Powered Nacelle (see Fig. 1). This fan was designed specifically for noise studies with a flexibility that allows variations in stator vane number and axial spacing. Except for a cylindrical section just downstream of the rotor (to accommodate the variable stator position) the external and internal inlet and exit contours simulate the flow path of the nacelle designed for JT9D engine installations on both the DC-10 and B-747 aircraft. The model fan can also be tested with a variety of inlet and aft duct designs of different contour and length. As shown in Fig. 1, the fan is powered by a single-stage turbine, which in turn is driven by compressed air or nitrogen.

The model fan was designed to operate at speeds up to 80,000 rpm, produce a fan pressure ratio of approximately 1.52, and thus simulate the performance of a full-scale fan in a typical high bypass ratio engine. It incorporates 18 blades which can be installed together with a 26 vane stator assembly to provide for "cut-on" BPF (blade passage frequency) interaction tone noise at fan tip speeds typical of aircraft approach conditions, or installed with a 40 vane stator assembly to provide a "cut-off" interaction at the same condition. The stator can be spaced downstream of the rotor 1.38, 2.63, or 3.88 axial rotor tip chords (Fig. 1) or the stator assembly can be completely removed. Since the Advanced Powered Nacelle does not incorporate large support struts, pylons, or core compressor stators, etc., near the fan, several tone noise sources that may exist in an actual engine are not simulated. However, valuable information can be and has been obtained on the nature of the interaction noise that is generated by rotor wakes impinging on the fan exit guide vane stators.

The Advanced Powered Nacelle was tested in the Acoustic Research Facility at United Technologies Research Center (see Fig. 2) to provide a carefully controlled, essentially turbulent-free inflow.

Figure 3 displays noise results at farfield angles of 60 and 120 deg to the inlet axis of the Advanced Powered Nacelle from a series of tests conducted at a fan speed of 56,000 rpm, duplicating the subsonic rotor tip speed of a typical high bypass ratio fan at aircraft approach powers.

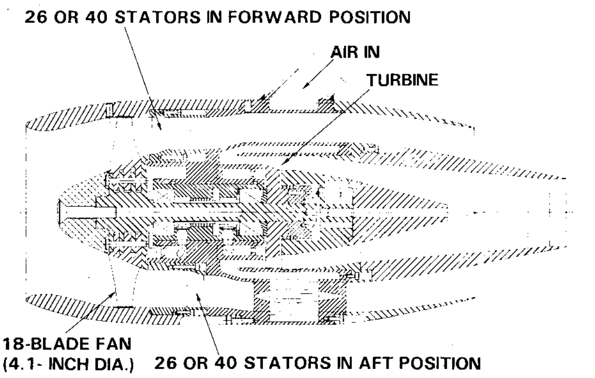

Fig. 1 Turbofan engine simulator (Advanced Powered Nacelle).

The result of specific interest here is the reduction of BPF tone noise (both inlet and aft) as a function of rotor-stator spacing for the 40 vane stator assembly. For this stator (together with the 18 blade rotor) the lowest interaction mode number at BPF is $m = 18 - 40 = -22$, which is well below cut-off at the 56,000 rpm speed shown. Consequently, noise at BPF should not be measured at this speed, let alone vary with rotor-stator spacing as shown. Although the tone level was much lower than that for the "cut-on" configuration (26 vanes), there still remains a significant farfield BPF tone whose level clearly does vary systematically with spacing. Thus, the source of noise would appear to be some form of rotor-stator interaction different from the nominal 22 lobe predicted pattern.

A possible explanation for the above behavior could be attributed to the small vane-to-vane irregularities that exist in the construction of the stator. Rotor wakes would interact with such an "asymmetric" stator to generate various small amounts of noise in all modes, and the contributions of the propagating portion of this set of "extraneous" modes would establish a basis for sound radiated to the farfield. An analysis of the noise generated by asymmetric (i.e., nonuniformly spaced) stators is presented in the following section. Numerical results and comparisons with data from the model fan tests are presented in Sec. IV, and full-scale engine applications are discussed in Sec. V.

III. Analysis

In the investigation of a related topic, the generation of extraneous frequencies by an imperfect rotor,[2] it was determined that deviations from uniform circumferential blade spacing were much more important than blade-to-blade variations in aerodynamic loading. The current analysis is consequently limited to examining the effects of departures from equal circumferential spacing of a set of stator vanes when interacting with the wakes from a symmetrical rotor.

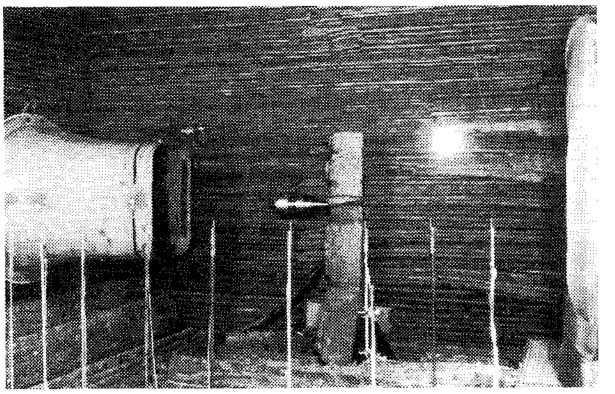

Fig. 2 Powered Nacelle in flight simulation Acoustic Research Facility.

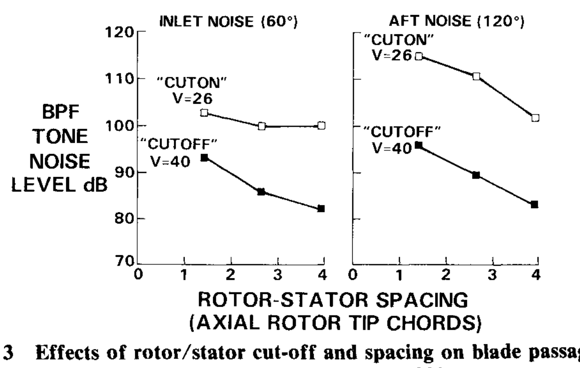

Fig. 3 Effects of rotor/stator cut-off and spacing on blade passage frequency tone levels from Powered Nacelle at 56,000 rpm.

The pressure field at a radial and axial location in the duct due to the interaction between the rotor and a single stator vane may be written:

$$p'(\theta,t) = Re\sum_m c_m \exp i(m\theta - nB\Omega t) \qquad (1)$$

It can then be shown[1] that the field resulting from the superposition of the effects of V vanes is

$$p(\theta,t) = Re\sum_m C_m \exp i(m\theta - nB\Omega t) \qquad (2)$$

where

$$C_m/c_m = \sum_{q=0}^{V-1} \exp ip_m\alpha_q \qquad (3)$$

$$p_m = nB - m \qquad (4)$$

and α_q is the angular location of the qth vane.

In the case of a symmetric stator where $\alpha_q = (2\pi/V)\,q$,

$$C_m/c_m = \sum_{q=0}^{V-1} \exp ip_m \frac{2\pi}{V} q$$

The complex sum vanishes unless p_m is a multiple $\pm k$ of V. When $p_m = \pm kV$ there follows $m = nb \pm kV$, which is the well-known rule[1] for determining the interaction modes having modal coefficients $C_m = Vc_m$.

The type of asymmetry to be now investigated is represented by letting the angles of α_q sustain independent random deviations about their nominal values, $\beta_q = (2\pi/V)q$. Then

$$\alpha_q = \beta_q + \epsilon_q = \frac{2\pi}{V} q + \epsilon_q \qquad (5)$$

The deviations ϵ_q will be taken as normally distributed about zero mean and with standard deviation σ_ϵ. (Note that the presence of the ϵ_q will introduce a plurality of propagating modes into a design that is cut-off when $\epsilon_q = 0$.) The analytical results follow two branches. First, the expected or mean value of the total acoustic power is obtained. This result is not restricted to "small" deviations, ϵ_q. The second branch of the work is based upon assuming that the deviations ϵ_q are small enough to allow the linearizations $\sin p_m\epsilon_q \approx p_m\epsilon_q$ and $\cos p_m\epsilon_q \approx 1$. This linearization makes it convenient to arrive at expressions for the probability density and distribution functions for total power so that the reliability of the expected value as a measure of total power can be assessed.

Expected Acoustic Power

To obtain the expected value of the total acoustic power, an expression must first be obtained for the expected value of the square of each of the modal coefficients, C_m. To obtain this, it is convenient to obtain the following form for $|C_m/c_m|^2$, based upon Eq. (3):

$$\begin{aligned}|C_m/c_m|^2 &= (C_m/c_m)(C_m/c_m)^* \\ &= \sum_{q=0}^{V-1} \exp ip_m(\beta_q+\epsilon_q) \sum_{j=0}^{V-1} \exp -ip_m(\beta_j+\epsilon_j) \\ &= \sum_{q=0}^{V-1}\sum_{j=0}^{V-1} [\exp ip_m(\beta_q-\beta_j)\exp ip_m(\epsilon_q-\epsilon_j)]\end{aligned} \qquad (6)$$

For the V terms where $q=j$ the exponents vanish so the contributions of these terms to the sum is V, giving

$$|C_m/c_m|^2 = V + \sum_{j\neq q} \exp ip_m(\beta_q-\beta_j)\exp ip_m(\epsilon_q-\epsilon_j)$$

Setting $\beta_q - \beta_j = \beta_{qj}$ and $\epsilon_q - \epsilon j = \epsilon_{qj}$ and noting that the $V^2 - V$ terms in the $j\neq q$ sum occur in complex conjugate pairs, we have:

$$\begin{aligned}|C_m/c_m|^2 &= V + \sum_{j>q}\left[\exp ip_m(\beta_{qj}+\epsilon_{qj}) + \exp -ip_m(\beta_{qj}+\epsilon_{qj})\right] \\ &= V + 2\sum_{j>q} \cos p_m(\beta_{qj}+\epsilon_{qj})\end{aligned}$$

The expected value can now be indicated:

$$E|C_m/c_m|^2 = V + 2\sum_{j>q} E\left[\cos p_m(\beta_{qj}+\epsilon_{qj})\right]$$

When the ϵ_q and ϵ_j are normally distributed with zero mean and variance σ_ϵ^2 it is found that

$$E(\cos p_m\epsilon_{qj}) = \exp(-p_m{}^2\sigma_\epsilon^2)$$

and

$$E(\sin p_m\epsilon_{qj}) = 0$$

The expected value then reduces to

$$E|C_m/c_m|^2 = V + 2\exp(-p_m{}^2\sigma_\epsilon^2)\sum_{j>q}\cos p_m\beta_{qj}$$

The sum can be shown to have the following value:

$$\sum_{j>q}\cos p_m\beta_{qj} = \tfrac{1}{2}\left[V^2\delta_{p_m v} - V\right]$$

where $\delta_{p_m v} = 1$ when p_m is an integer multiple of V, and $\delta_{p_m v} = 0$ otherwise:

This gives the result

$$E|C_m/c_m|^2 = V + (\delta_{p_m v}V^2 - V)\exp(-p_m{}^2\sigma_\epsilon^2) \qquad (7)$$

It is more convenient to describe separately the "extraneous modes" given by $m\neq nB\pm kV$, which would be absent in a symmetric stator and the "nominal interactions" that would occur in a symmetric stator interaction when $m = M = nB\pm kV$:

1) Extraneous modes $m\neq nB\pm kV$:

$$E|C_m/c_m|^2 = V\left[1-\exp(-p_m{}^2\sigma_\epsilon^2)\right] \qquad (8)$$

This has the limiting values

$$E|C_m/c_m|^2 \xrightarrow[\sigma_\epsilon\to 0]{} Vp_m{}^2\sigma_\epsilon^2 \qquad (8a)$$

$$E|C_m/c_m|^2 \xrightarrow[\sigma_\epsilon\to\infty]{} V \qquad (8b)$$

2) Normal interaction, $m=M=nB\pm kV, p_m=p_M=\pm kV$:

$$E|C_m/c_m|^2=E|C_M/c_M|^2=V^2-(V^2-V)\left[1-\exp(-p_M\sigma_\epsilon^2)\right] \quad (9)$$

with limiting values

$$E|C_M/c_M|^2 \xrightarrow[\sigma_\epsilon\to 0]{} V^2-(V^2-V)p_M{}^2\sigma_\epsilon^2 \quad (9a)$$

$$E|C_M/c_M|^2 \xrightarrow[\sigma_\epsilon\to \infty]{} V \quad (9b)$$

With the above expressions, the sound power can now be calculated. The sound power flux in a constant area annular duct is given by

$$W=\frac{1}{2\rho a}\sum_{m\mu} a_{m\mu}C_{m\mu}C^*_{m\mu}A_{m\mu} \quad (10)$$

where

$$a_{m\mu}=\sqrt{1-1/\xi^2_{m\mu}} \quad (11)$$

$\xi_{m\mu}$ is the cut-off ratio and $A_{m\mu}$ is the effective area for the (m,μ) mode.

In the preceding analysis of the effect of random angular spacing variations, the radial variations of pressure have been ignored. To conform with the limitations of this simplification, the above expression for power will be modified to the following:

$$W'=\sum_m a_m|C_m|^2$$

where the factors $A_{m\mu}/2\rho a$ are all taken as unity and it is supposed that a single C_m suffices to specify the pressure of each m-lobe pattern. For purposes of the specific calculations that follow it will also be assumed that the modal coefficients c_m giving the interaction field of a single stator vane are all equal ($c_m=c$). Then the following approximation will be used for "normalized" power:

$$\text{normalized power}=U=\sum_m a_m|C_m/c|^2 \quad (12)$$

This expression, incorporating the several simplifications noted above, will not generally suffice for computing acoustic power accurately, but it is taken as a reasonable approximation for the comparative type of calculations that are to be made presently.

The expected value of the total normalized power resulting from all the extraneous modes then becomes:

$$E(U)=\sum_m a_m E|C_m/c|^2$$

$$=V\sum_m a_m\left[1-\exp(-p_m{}^2\sigma_\epsilon^2)\right] \quad (13)$$

when σ_ϵ is sufficiently small and, recalling that $p_m=nB-m$, this simplifies to:

$$E(U)=\sigma_\epsilon^2 V\sum_m a_m(nB-m)^2 \quad (14)$$

The summation is restricted to (positive and negative) values of m associated with propagating modes for which a_m, given by Eq. (11), is real.

This basic result is examined in more detail in the next section. First, it is desirable to obtain the probability density and distribution functions for the total normalized power U in order to see how well $E(U)$ provides a measure of the total power of the extraneous modes.

Probability Density and Distribution Functions

The probability density function and its integral, the distribution function, give the probability distribution as a function of the magnitude of the normalized power. From these functions can be determined, for example, the probability that the extraneous modal power will exceed its mean or expected value by any specified amount.

Derivation of these results is more lengthy than for the expected power and is given in the Appendix. In brief, it is found that the individual extraneous modal amplitudes each has a Rayleigh distribution and that their squares are exponentially distributed. These properties are used to arrive at the following expressions (see Appendix) giving the probability density and distribution functions for total acoustic power.

1) Probability density function for power:

$$h(U)=\tfrac{1}{2}\sum_i\left\{\frac{(\sigma_i'^2)^{N-2}}{\prod_{j\neq i}(\sigma_i'^2-\sigma_j'^2)}\exp\left(-\frac{1}{2\sigma_i'^2}U\right)\right\} \quad (15)$$

2) Probability distribution function:

$$H(U)=\int_{-\infty}^{u}h(U)dU=1-\sum_i\left\{\frac{(\sigma_i'^2)^{N-1}}{\prod_{j\neq i}(\sigma_i'^2-\sigma_j'^2)}\exp\left(-\frac{1}{2\sigma_i'^2}U\right)\right\} \quad (16)$$

where

$\sigma_m'^2=a_m\sigma_{p_m}^2=a_m\tfrac{1}{2}Vp_m^2\sigma_\epsilon^2$

$p_m=nB-m$

i,j = mode indices ranging from $m_{\min}$ to $m_{\max}$

N = number of propagating modes

Numerical evaluation of these functions provides a measure of the spread or dispersion of the acoustic power about the expected value $E(U)$ given by Eqs. (13) or (14) and is contained in the next section.

IV. Numerical Results and Experimental Comparisons

Farfield blade passage noise radiation patterns were obtained for the model fan described in Sec. II, equipped with cut-off and cut-on stator assemblies, each tested at two speeds. The 26 vane stator was selected for the reference configuration, and was calculated to produce a cut-on interaction with the 18 blade fan at blade passage frequency ($m=18-26=-8$) at the test speeds of 40,000 and 56,000 rpm. The other stator employed 40 vanes and at these speeds was predicted to produce a highly decaying interaction ($m=18-40=22$) at blade-passage frequency.

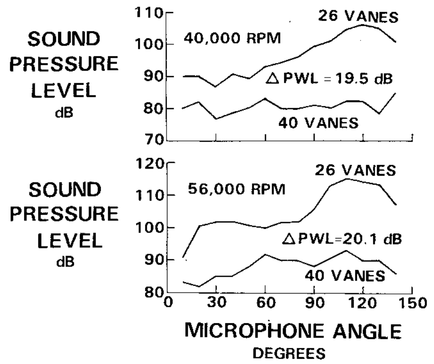

Fig. 4 Blade passage frequency directivity patterns (4.1 in. diameter model fan).

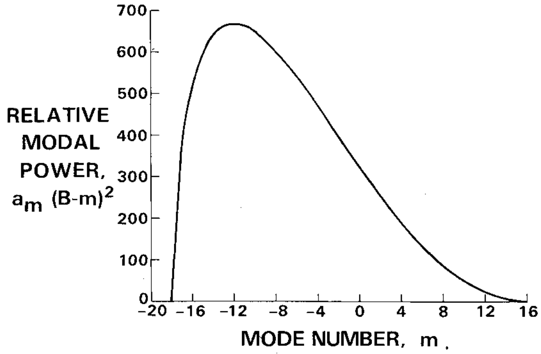

Fig. 5 Distribution of expected modal power (4.1 in. fan, $V=40$, 56,000 rpm).

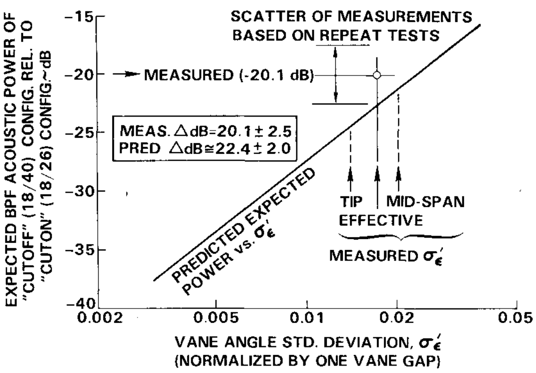

Fig. 6 Blade passage frequency tone power due to stator asymmetry (predicted and measured for Powered Nacelle at 56,000 rpm).

Figure 4 displays the measured blade passage frequency (BPF) radiation patterns for these configurations at two speeds. From these patterns it is found that the acoustic power of the cut-off 40 vane configuration is about 20 dB lower than that of the reference cut-on 26 vane stator configuration. At 40,000 rpm the 40 vane relative level is -19.5 dB and at 56,000 rpm it is -20.1 dB. (These differences are subject to some uncertainty because of limits on the repeatability of the data, but repeatability tests indicate that the above differences should be repeatable to within ± 2.5 dB.)

In principle, of course, BPF noise should not be detectable from tests of the 40 vane fan if construction and test conditions were ideal. The objective of this analysis is to determine if deviations from perfectly uniform circumferential positioning of the stator vanes can account for the measured levels of about -20 dB relative to the 26 vane reference configuration.

The expected value of acoustic power of the 40 vane configuration was computed using Eq. (14), retaining the vane deviation σ_ϵ as a parameter. At 56,000 rpm the

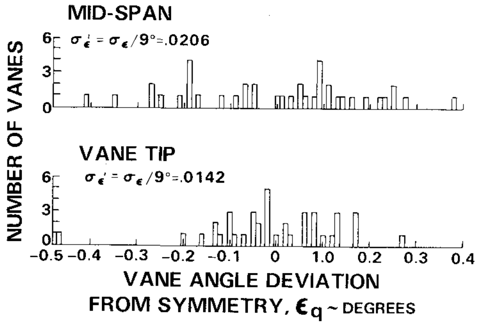

Fig. 7 Measured vane angle variations for 40 vane stator.

propagating modes are limited to values of m between ± 17. Figure 5 shows the relative magnitudes of the components, $a_m\ (B-m)^2$, whose sum is proportional to the normalized power. Total power is computed and referenced to the power calculated for the 26 vane build. This reference power is obtained for the propagating -8 lobe pattern by taking $U_{\rm ref} = a_{-8}\ V^2 = (26)^2\ a_{-8}$. An additional assumption is consequently introduced, i.e., that the modal coefficients c due to a single vane interaction are the same in both 26 and 40 vane stators.

Figure 6 shows at 56,000 rpm the expected power relative to that of the 26 vane stator as a function of the vane spacing standard deviation in the 40 vane configuration. For convenience, a value of deviation σ_ϵ' normalized to one vane spacing is used:

$$\sigma_\epsilon{}' = \sigma_\epsilon / (2\pi/V)$$

To allow comparison between measured and calculated values of power it is necessary to obtain a value of σ_ϵ' for the 40 vane stator. This was done by measuring the individual vane location angles near the tip and also at midspan, using an optical comparator. Values of the deviations ϵ_q about equal spacing are shown in Fig. 7.

The corresponding normalized standard deviations are tabulated in Table 1 together with the rms average. These values allow Fig. 6 to be entered. The expected power ranges from -24.3 dB using the tip deviation to -21.1 dB for the midspan value. Using the rms average deviation gives an

Table 1 Normalized standard deviations

Location	Standard deviation, $\sigma_\epsilon{}' = \sigma_\epsilon/(2\pi/40)$
Tip	0.0142
Midspan	0.0206
rms average	0.0177

Table 2 Expected vs measured power at 56,000 rpm

Vane deviation for comp. expected power	Expected power, dB	Measured power, dB	Difference, dB
Tip	−24.3	−20.1(±2.5)	4.2
Midspan	−21.1	−20.1(±2.5)	1.0
rms average	−22.4	−20.1(±2.5)	2.3

Table 3 Power calculations at 56,000 rpm

Location	Power based on actual angles, dB	Expected power using standard deviation, dB	Difference, dB
Tip	−26.0	−24.3	1.7
Midspan	−22.5	−21.1	1.4

expected power of −22.4 dB with respect to the 26 vane reference stator. Also shown on Fig. 6 is the measured power difference between these configurations (−20.1 dB) and the possible ±2.5 dB scatter due to data repeatability.

Thus the calculated expected power is roughly 2 dB lower than measured. Results at the other test speed (40,000 rpm) did not agree as closely, the expected power being about 4 dB lower than measured. Table 2 summarizes results at 56,000 rpm.

An additional calculation was made to assess further the validity of the expected power method. Since the individual angles α_q of the 40 vane stator were measured to obtain their standard deviation, it was decided to use these actual angles to compute coefficient ratios C_m/c using Eq. (3). On this basis the power calculations for 56,000 rpm, (still using the 26 vane as reference) produced the comparison shown in Table 3. These figures indicate that the procedure of using the expected power based upon statistical methods provides a good approximation to the calculated total power of the extraneous modes based upon detailed knowledge of individual vane spacing, despite the fact that the spacing deviations plotted in Fig. 7 depart significantly from a Gaussian distribution. Differences between the power in any one random build and the expected power are put in perspective by using the probability density and distribution functions, now to be discussed.

In the next section the expected power method will be used to estimate the practical benefits of providing cutoff stator designs in full-scale engines. Before considering that topic it is necessary to determine how good a measure of power is provided by the expected value. The answer lies in examination of the probability density and distribution functions which provide an indication of what departures from the expected or mean value are reasonably likely to occur.

The probability density function for extraneous modal power was computed using a modified form of Eq. (15) for the 40 vane stator at a shaft speed of 56,000 rpm. Instead of using normalized power, the independent variable is taken as the ratio of normalized power to its expected value. This procedure makes the independent variable quantification easier to identify on a familiar basis, and also frees the result from dependence upon the vane angle deviation. Figure 8 illustrates the resulting density function. The compactness of the distribution should be noted. This is a consequence of the large number (35) of terms contributing to the power; as this number increases the distribution approaches a δ function.

A comparison figure (Fig. 9) shows the complement $H'(U)$ of the distribution function of Eq. (16). The complement gives the probability that the power will exceed the expected value by a given amount. This function is of greater practical utility than the density function—for by subtracting $H'(U_1) - H'(U_2)$ the probability of the power lying between any values, U_1 and U_2, is obtained directly. As with the density function, the power is expressed relative to the expected value.

From the figures represented in Fig. 9, Table 4 indicates the compactness of the distribution of total extraneous power. From this table in particular it can be seen that the probability of the power exceeding its expected value by 2 or more dB is trivially small for practical purposes. Accordingly, we can use expected values with some confidence in estimating the effects of stator imperfections in limiting benefits of cut-off stator designs for full-scale engines.

Table 4 Distribution of total extraneous power

		Probability
Power deviation range from expected value, dB	±0.5	0.414
	±1.0	0.723
	±1.5	0.896
	±2.0	0.968
Power excess over expected value, dB	0 or more	0.469
	+0.5 or more	0.264
	+1.0 or more	0.114
	+1.5 or more	0.036
	+2.0 or more	0.008

V. Full-Scale Applications

The preceding results for a model 40 vane fan stator show that nominal spacing irregularities in a cut-off stator design will allow very substantial noise reduction with respect to a reference cut-on design.

For a standard deviation in cut-off vane spacing error of 1%, more than 25 dB noise reduction is obtained relative to the cut-on case. It is important to verify that comparable benefits are obtainable in full-scale fans. Accordingly, calculations were made for a fan employing a 46 blade rotor and a 108 vane stator (a blade/vane number combination more representative of current high bypass engine designs). The 46 − 108 = −62 lobe BPF interaction is cut off. Values of expected power were determined for random spacing deviations. To provide a basis for comparison, hypothetical stators of 50, 60, and 70 vanes were used for reference powers of what conceivably might be used in cut-on stator designs.

The results show that for an assumed 1% deviation in the 108 vane stator angles, noise reductions of 26-28 dB are obtained with respect to these cut-on stators, showing that the noise generation is not particularly sensitive to blade/vane numbers. It may therefore be concluded that for cut-off stators having manufacturing tolerances of less than 1% of vane circumferential spacing, the residual noise levels are substantially lower than for cut-on stator designs. Under certain operating conditions other sources of fan noise, such as caused by inflow irregularities, may mask the low noise

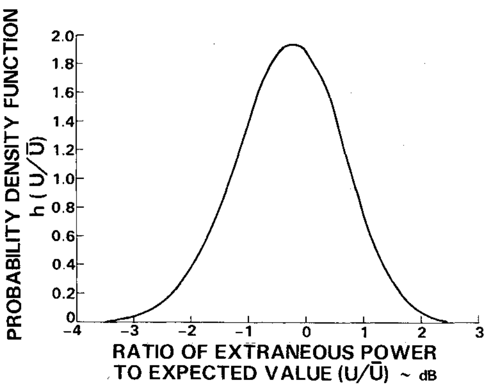

Fig. 8 Probability density function for extraneous power (model fan, 40 vanes, 56,000 rpm).

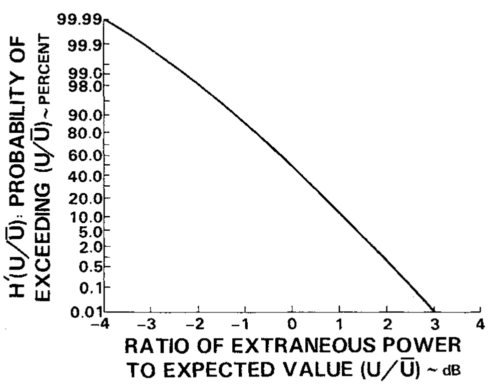

Fig. 9 Probability distribution function for extraneous power (model fan, 40 vanes, 56,000 rpm).

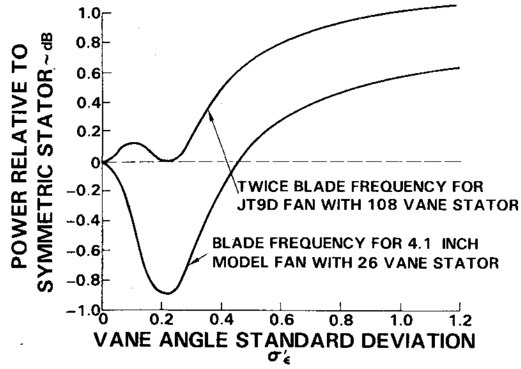

Fig. 10 Acoustic power change for cut-on stators with vane asymmetry.

levels produced by typical cut-off stator designs. (The special test conditions applicable to the model fan experiments described here are not representative in this regard.)

One final question is examined: Is it possible to achieve a noise reduction in a cut-on stator by the deliberate introduction of large random spacing variations? Calculations of expected power were made for the blade passage frequency interaction of the model 26 vane stator and also for the interaction at twice blade passage frequency of the 108 vane JT9D stator. (The exact forms, Eqs. (8) and (9), were used to allow effects of large deviations to be evaluated.) In each case the reference power was calculated for the corresponding stator with perfect spacing.

These results are shown in Fig. 10. In the 26 vane case, a very small (less than 1 dB) reduction was indicated at around 20% spacing deviation; for greater variations, the presence of many extraneous modes results in noise levels about 1 dB higher than for the symmetrical case. For the 108 vane stator no reduction was predicted. (Although calculations were made for a very wise range of σ_ϵ', clearly no practical stator could operate with deviations in excess of a small fraction of nominal spacing.) In both cases no worthwhile reductions can be predicted by spacing deviations. It is therefore unlikely that the concept of using an asymmetric arrangement of a nominally cut-on stator can be used as an effective noise reduction measure.

VI. Summary

A probability analysis has been made of the effect upon noise of angular spacing variations in the vanes of a stator producing a nominally cut-off interaction. Comparison of calculations with measurements of a model fan run in an acoustic wind tunnel indicate that the method provides a reasonable quantitative estimate of the residual or extraneous rotor/asymmetric stator interaction noise in a nominally cut-off configuration. It is concluded that full-scale cut-off configurations incorporating nominal manufacturing tolerances will generate very significantly less noise than comparative cut-on designs. Use of intentional spacing asymmetry as a noise reduction concept for cut-on stator configurations appears to have no practical value.

Appendix

The probability density function for the total normalized power of the extraneous modes ($m \neq nB \pm kV$) will be found by deriving the density functions for $|C_m/c|$, $|C_m/c|^2$, $a_m |C_m/c|^2$, and finally their sums.

Transforming Eq. (3) into rectangular form, expanding and linearizing by use of $\sin p_m \epsilon_q = p_m \epsilon_q$, $\cos p_m \epsilon_q = 1$ gives, the extraneous mode amplitudes:

$$\frac{C_m}{c} = -p_m \sum_{q=0}^{V-1} \epsilon_q \sin p_m \beta_q + i p_m \sum_{q=0}^{V-1} \epsilon_q \cos p_m \beta_q \qquad \text{(A1)}$$

If the ϵ_q are normally distributed with zero mean and standard deviation σ_ϵ, then the real and imaginary parts of C_m/c will each be normally distributed about zero with a common variance:

$$\sigma_{p_m}^2 = \tfrac{1}{2} p_m{}^2 \sigma_\epsilon^2 V \qquad \text{(A2)}$$

(For stators with an even number of vanes there is a sparse set of exceptions to Eq. (A2): When p_m is an integer multiple of *one-half V*, the modal coefficient ratio is purely imaginary and its variance is twice the value indicated by Eq. (A2). These cases are not encountered in the current applications of this analysis and are consequently ignored.)

It can also be shown from Eq. (A1) that the real and imaginary parts of C_m/c are linearly uncorrelated. The above conditions are the requirements that the amplitude of each of the modal coefficient ratios has a probability density function characterized as a Rayleigh distribution:

$$f(r_m) = f(|C_m/c|) = (r_m/\sigma_{p_m}^2)\exp(-r_m^2/2\sigma_{pm}^2) \qquad \text{(A3)}$$

To obtain the density functions $g(\rho_m)$ for the square of the amplitude, $\rho_m = r_m{}^2$, we use the standard transformation for density functions,

$$g(\rho_m) = f(r_m)\,|\mathrm{d}r_m/\mathrm{d}\rho_m| \qquad \text{(A4)}$$

getting the exponential distribution

$$g(\rho_m) = \frac{1}{2\sigma_{p_m}^2}\exp(-\rho_m/2\sigma_{p_m}^2) \qquad \text{(A5)}$$

For each term, $\rho_m{}' = a_m |C_m/c|^2$ in the normalized power summation Eq. (12), the density function called $g'(\rho m')$ follows:

$$g'(\rho_m{}') = g'(a_m\rho_m) = g(\rho_m)\,|\mathrm{d}\rho_m/\mathrm{d}\rho_m{}'| = (1/a_m)g(\rho_m)$$

$$= (1/2\sigma_m'{}^2)\exp-(\rho_m{}'/2\sigma_m'{}^2) \qquad \text{(A6)}$$

where

$$\sigma_m'{}^2 = a_m\sigma_{p_m}^2 = \tfrac{1}{2} a_m p_m{}^2 \sigma_\epsilon^2 V \qquad \text{(A7)}$$

For normalized power given by the sum of the $\rho_m{}' = a_m |C_m/c|^2$ it is recalled that the probability density function for the sum $U = x_i + x_j$ of two independent variables with density functions $f_i(x_i)$, $f_j(x_j)$ is the convolution:

$$h(U) = h(x_i + x_j) = f_i \star f_j = \int_{-\infty}^{\infty} f_i(U - x_j) f_j(x_j)\,\mathrm{d}x_j$$

or

$$\int_{-\infty}^{\infty} f_j(U - x_i) f_i(x_i)\,\mathrm{d}x_i \qquad \text{(A8)}$$

Thus the probability density for the normalized power,

$$U = \sum^{N} \rho_m{}' = \sum^{N} a_m |C_m/c|^2$$

designated as $h(U)$ is

$$h(U) = \rho_1' \quad \rho_2' \star \ldots \rho_N' \;\star \quad (N \text{ is number of terms})$$

Applying Eq. (A8) $N-1$ times and simplifying generates the resulting density function for power Eq. (15) and straightforward integration then gives the corresponding distribution function $H(U)$ of Eq. (16).

References

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[2] Pickett, G.F., "The Prediction of the Spectral Content of Combination Tone Noise," AIAA Paper 71-730, June 1971.